

AD-A171 031

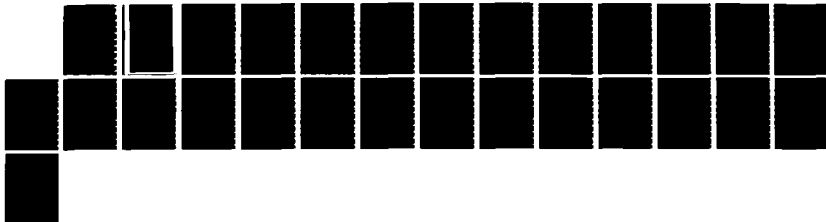
EXAMINATION OF A TECHNIQUE FOR THE EARLY DETECTION OF  
FAILURE IN GEARS BY.. (U) AERONAUTICAL RESEARCH LABS  
MELBOURNE (AUSTRALIA) P D MCFADDEN APR 86  
ARL/AERO-PROP-TN-434

1/1

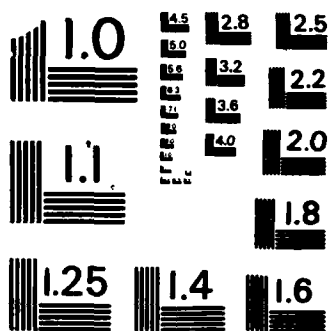
UNCLASSIFIED

F/G 13/9

NL







MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A



12

AD-A171 031



**DEPARTMENT OF DEFENCE**  
**DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION**  
**AERONAUTICAL RESEARCH LABORATORIES**  
**MELBOURNE, VICTORIA**

Aero Propulsion Technical Memorandum 434

**EXAMINATION OF A TECHNIQUE FOR THE EARLY DETECTION  
OF FAILURE IN GEARS BY SIGNAL PROCESSING OF THE  
TIME DOMAIN AVERAGE OF THE MESHING VIBRATION**

by

P.D. McFADDEN

Approved for Public Release

AUG 0 1986

DTIC FILE COPY

(C) COMMONWEALTH OF AUSTRALIA 1986

APRIL 1986



DEPARTMENT OF DEFENCE  
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION  
AERONAUTICAL RESEARCH LABORATORIES

Aero Propulsion Technical Memorandum 434

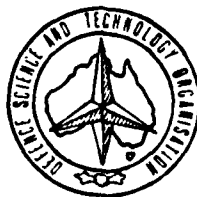
EXAMINATION OF A TECHNIQUE FOR THE EARLY DETECTION OF FAILURE IN  
GEARS BY SIGNAL PROCESSING OF THE TIME DOMAIN AVERAGE OF THE  
MESHING VIBRATION

by

P.D. McFadden

SUMMARY

An existing technique for the signal processing of the time domain average of the tooth meshing vibration of gears is examined with application to the early detection of failure. It is shown that the "regular" signal extracted from the time domain average of the gear vibration by the elimination of the fundamental and harmonics of the tooth meshing frequency forms the time domain average of the vibration of a single tooth. The "residual" signal which is obtained by subtracting the regular signal from the original time domain average gives the departures of the vibration from the average. Numerical and practical examples are given.



© COMMONWEALTH OF AUSTRALIA 1986

---

POSTAL ADDRESS: Director, Aeronautical Research Laboratories,  
P.O. Box 4331, Melbourne, Victoria 3001, Australia.



## CONTENTS

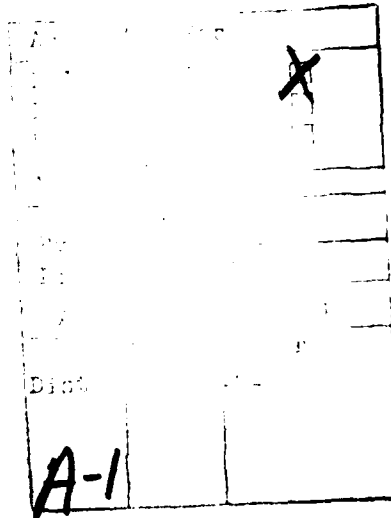
<u>Section</u>	<u>Page No</u>
1. INTRODUCTION	1
2. THEORETICAL DEVELOPMENT	2
3. NUMERICAL EXAMPLES	7
4. PRACTICAL EXAMPLES	9
5. CONCLUSIONS	10

REFERENCES

FIGURES

DISTRIBUTION

DOCUMENT CONTROL DATA





## 1. INTRODUCTION

The early detection of incipient failure in mechanical systems is of great practical importance to operators as it permits the scheduled shutdown and repair of a system instead of unexpected catastrophic failure. There are several techniques available for the early detection of failure, and one of the most useful of these is vibration analysis. The underlying premise of vibration analysis is that a change in the mechanical condition of a system may produce a change in the vibration which the system produces. In extremely simple systems, this change may take the form of an increase in the amplitude of the total vibration which can be readily detected with simple instruments. For more complex systems, changes in the total vibration due to deterioration of a single machine element will be less significant, and more sophisticated techniques may be needed if the damage is to be identified.

A popular technique of vibration analysis during the last decade has been spectral analysis, in which the amplitude spectrum of the measured vibration time signal is calculated and displayed. It is a particularly powerful technique because the different elements of a mechanical system in general produce vibration at different frequencies. For example, gearboxes produce vibration at the fundamental and harmonics of the gear tooth meshing frequencies, which are usually different for each pair of gears in the gearbox. Thus changes in the spectrum, such as an increase in the amplitude of the vibration at a particular frequency, can often be traced to a single element in a complex system by the frequency at which the change occurs. But this identification can be difficult in very complex systems such as helicopter gearboxes, which may have as many as 30 gears and 50 bearings. The great number of spectral lines which these elements produce during normal operation can make it difficult to even detect changes in the spectrum, let alone identify the source, particularly as these changes may at first be subtle and require careful comparison with the spectrum of the same system in good condition if they are to be detected.

An alternative technique of vibration analysis which is becoming more commonly used for early detection of failure in gears is called time domain averaging. If a second signal is acquired which is synchronized with the rotation of the gear of interest, and the ensemble average of the vibration is calculated with the start of each frame being determined by the synchronizing signal, then it is found that after many averages all of the vibration which is asynchronous with the rotation of that gear tends to cancel, leaving an estimate of the vibration of the gear of interest during one revolution. This technique is particularly useful for complex systems such as gearboxes as it eliminates the vibration from other system elements, thus reducing the problem to the study of a simpler system.

Whereas the amplitude spectrum is presented in the frequency domain, the time domain average, as its name implies, is presented in the time domain, making it possible to compare the vibration produced by different teeth on the same gear. As many failure modes involve the deterioration of only a part of a gear, it should be possible to identify the affected part by comparison with the remaining sound part of the gear, and so, in principle, should enable the detection of many failure modes on the basis of a single vibration measurement. This would be a great advantage over spectrum analysis which, because of the complexity of the spectrum, invariably requires comparison of the suspect spectrum with the spectrum of



the same or a similar system in good condition.

If sufficient averages are taken then the time domain average closely approximates a truly periodic signal, with the very important result that the Fourier transform of the time domain average is a pure line spectrum. This enables the manipulation of the time domain average in the frequency domain, including such powerful operations as ideal filtering. These operations are not possible with conventional spectral analysis because there are many frequency components present in the incoming signal which are not in general exactly periodic within the block of data which is sampled, making it necessary to multiply the data block by a window function which tapers the values at the ends of the block to zero in order to force periodicity. Unfortunately, this broadens the spectral lines and can cause nearby lines to overlap, rendering the manipulation of the spectrum difficult or impossible.

Why perform this manipulation? The changes in the vibration of a gear which are produced when a defect is small may also be small, and may not be readily detected against the normal vibration of the gear. It is possible to enhance the clarity of the changes in the time domain average by digital signal processing, using techniques which remove the normal vibration from the time domain average so that the changes in the vibration are more readily apparent. In one of these enhancement techniques, pioneered by Stewart [1], all of the tooth meshing components and their harmonics are eliminated from the spectrum of the time domain average and the remaining time signal reconstructed to produce what is called the "residual" signal. It has been demonstrated that this signal often shows evidence of a defect before it can be seen in the time domain average. The components which are eliminated constitute what Stewart calls the "regular" signal. The technique can be extended to include the removal of the low order modulation sidebands about the meshing components, but only the simplest form will be considered here.

But even though the technique has been in use for many years, there has been no satisfactory quantitative explanation of what the regular and residual signals actually represent. While other enhancement techniques have recently been developed [2-4], these are based on the signal processing of a narrow band of the spectrum of the time domain average, and an explanation for Stewart's broad band technique is still awaited. This technical memorandum examines Stewart's enhancement technique in its simplest form and shows, using simple theory, that the tooth meshing harmonics which Stewart refers to as the regular components actually define the time domain average of the meshing vibration of a single gear tooth, and so can be considered as the average tooth meshing vibration. The residual signal therefore defines the departure of the actual meshing vibration from the average. These findings are demonstrated by the analysis of several numerically generated signals and of the meshing vibration of a gear with a fatigue crack in one of the teeth.

## 2. THEORETICAL DEVELOPMENT

Consider a pair of meshing gears which operate at a constant speed and constant load, and which have different numbers of teeth. Assume initially that the teeth on each gear are identical and are free of pitch errors, and that the gears are free of eccentricity. Hence the tooth meshing vibration



can be represented by a finite Fourier series with a fundamental frequency which is equal to the tooth meshing frequency  $f_x$ , given by the product of the number of teeth  $Z$  on the first gear and its rotational frequency  $f_r$ :

$$x(t) = \sum_{m=0}^M X_m \cos(2\pi m f_x t + \chi_m) \quad (1)$$

Now consider the case in which the first gear of the meshing pair has teeth which are nonuniform in profile or in pitch, or else the gear is eccentric, thereby producing amplitude and phase modulation of the tooth meshing vibration. Because the second gear remains unchanged, all modulation will be periodic with the rotation of the first gear at the frequency  $f_r$ . The amplitude and phase modulation functions  $a_m(t)$  and  $b_m(t)$  respectively can thus be represented by finite Fourier series in  $f_r$ . Note that these modulation functions may vary from one meshing harmonic to the next, so the subscript  $m$  must be incorporated in the equations:

$$a_m(t) = \sum_{n=0}^N A_{mn} \cos(2\pi n f_r t + \alpha_{mn}) \quad (2)$$

$$b_m(t) = \sum_{n=0}^N B_{mn} \cos(2\pi n f_r t + \beta_{mn}) \quad (3)$$

The modulated gear meshing vibration is given by:

$$y(t) = \sum_{m=0}^M X_m (1 + a_m(t)) \cos(2\pi m f_x t + \chi_m + b_m(t)) \quad (4)$$

Now consider the simple case in which the phase modulation is very small. If  $|b_m(t)| \ll \pi/2$ , then  $\cos(b_m(t)) \approx 1$  and  $\sin(b_m(t)) \approx b_m(t)$ , these being known as the small angle approximations. Thus for  $|b_m(t)| \ll \pi/2$  the following simplification can be made:

$$\begin{aligned} \cos(2\pi m f_x t + \chi_m + b_m(t)) &= \cos(2\pi m f_x t + \chi_m) \cos(b_m(t)) - \sin(2\pi m f_x t + \chi_m) \sin(b_m(t)) \\ &\approx \cos(2\pi m f_x t + \chi_m) - b_m(t) \sin(2\pi m f_x t + \chi_m) \end{aligned} \quad (5)$$

Substituting Equation 5 into Equation 4 yields:

$$\begin{aligned} y(t) &= \sum_{m=0}^M X_m (1 + a_m(t)) [\cos(2\pi m f_x t + \chi_m) - b_m(t) \sin(2\pi m f_x t + \chi_m)] \\ &= \sum_{m=0}^M X_m [\cos(2\pi m f_x t + \chi_m) - b_m(t) \sin(2\pi m f_x t + \chi_m) \\ &\quad + a_m(t) \cos(2\pi m f_x t + \chi_m) - a_m(t) b_m(t) \sin(2\pi m f_x t + \chi_m)] \end{aligned} \quad (6)$$

Furthermore, if  $|a_m(t)b_m(t)| \ll 1$  this equation can be simplified yet again to give:



$$\begin{aligned}
 y(t) &= \sum_{m=0}^M X_m [\cos(2\pi m f_x t + \chi_m) + a_m(t) \cos(2\pi m f_x t + \chi_m) \\
 &\quad - b_m(t) \sin(2\pi m f_x t + \chi_m)] \\
 &= x(t) + \sum_{m=0}^M X_m [a_m(t) \cos(2\pi m f_x t + \chi_m) - b_m(t) \sin(2\pi m f_x t + \chi_m)] \quad (7)
 \end{aligned}$$

Substituting Equations 1, 2 and 3 into Equation 7 yields:

$$\begin{aligned}
 y(t) &= \sum_{m=0}^M X_m [\cos(2\pi m f_x t + \chi_m) \\
 &\quad + \sum_{n=0}^N A_{mn}/2 (\cos(2\pi(m f_x - n f_R)t + \chi_m - \alpha_{mn}) \\
 &\quad \quad \quad + \cos(2\pi(m f_x + n f_R)t + \chi_m + \alpha_{mn})) \\
 &\quad - \sum_{n=0}^N B_{mn}/2 (\sin(2\pi(m f_x - n f_R)t + \chi_m - \beta_{mn}) \\
 &\quad \quad \quad + \sin(2\pi(m f_x + n f_R)t + \chi_m + \beta_{mn}))] \quad (8)
 \end{aligned}$$

Thus the modulated signal will comprise the original unmodulated signal, plus additional components at the tooth meshing harmonics, together with N pairs of modulation sidebands about each meshing harmonic, with the sidebands being spaced at multiples of the shaft rotation frequency  $f_R$  of the first gear.

The time domain average of a signal is calculated by the convolution of the original signal in the time domain with a train of ideal impulses, a process which is equivalent to the multiplication in the frequency domain of the Fourier transform of the original signal by a comb filter [5]. If sufficient averages are taken, the comb filter can be approximated by a train of ideal impulses located at multiples of the repetition frequency, so that multiplication of the Fourier transform of the original signal by this comb filter leaves only components at multiples of the repetition frequency. For the analysis of gear vibration, the gear rotation frequency  $f_R$  can be selected as the desired repetition frequency, in which case the time domain average  $c(t)$  will contain only components at multiples of the gear rotation frequency, namely all the tooth meshing harmonics and all of the modulation sidebands from the first gear. But for the system described here, Equation 8 has shown that there are no other components present, therefore  $c(t)=y(t)$ .

If the number of pairs of modulation sidebands N is such that  $N < Z$ , then the bandwidth  $2Nf_R$  occupied by the sidebands about each meshing harmonic will not extend as far as the adjacent meshing harmonics. Therefore the components of  $y(t)$  which contribute to  $c(t)$  at any given meshing harmonic  $m=K$  having a frequency  $f=Kf_x$  will be given by:

$$\begin{aligned}
 c_K(t) &= X_K \cos(2\pi K f_x t + \chi_K) \\
 &\quad + A_{K0} X_K/2 [\cos(2\pi K f_x t + \chi_K - \alpha_{K0}) + \cos(2\pi K f_x t + \chi_K + \alpha_{K0})] \\
 &\quad - B_{K0} X_K/2 [\sin(2\pi K f_x t + \chi_K - \beta_{K0}) + \sin(2\pi K f_x t + \chi_K + \beta_{K0})] \quad (9)
 \end{aligned}$$



However, if the number of pairs of modulation sidebands  $N$  is such that  $N > Z$ , then the bandwidth occupied by the sidebands will overlap the adjacent meshing harmonics. Thus the harmonic at  $f = Kf_x$  will coincide with the upper sideband of the next lower harmonic at  $f = (K-1)f_x + Zf_r = Kf_x$ , and with the lower sideband of the next higher harmonic at  $f = (K+1)f_x - Zf_r = Kf_x$ , with the result being given by the vector sum of the coinciding components. Therefore the components of  $c(t)$  at  $f = Kf_x$  will be:

$$\begin{aligned} c_K(t) = & X_K \cos(2\pi Kf_x t + \chi_K) \\ & + A_{K0} X_K / 2 [\cos(2\pi Kf_x t + \chi_K - \alpha_{K0}) + \cos(2\pi Kf_x t + \chi_K + \alpha_{K0})] \\ & - B_{K0} X_K / 2 [\sin(2\pi Kf_x t + \chi_K - \beta_{K0}) + \sin(2\pi Kf_x t + \chi_K + \beta_{K0})] \\ & + A_{K-1Z} X_{K-1} / 2 \cos(2\pi [(K-1)f_x + Zf_r] t + \chi_{K-1} + \alpha_{K-1Z}) \\ & - B_{K-1Z} X_{K-1} / 2 \sin(2\pi [(K-1)f_x + Zf_r] t + \chi_{K-1} + \beta_{K-1Z}) \\ & + A_{K+1Z} X_{K+1} / 2 \cos(2\pi [(K+1)f_x - Zf_r] t + \chi_{K+1} - \alpha_{K+1Z}) \\ & - B_{K+1Z} X_{K+1} / 2 \sin(2\pi [(K+1)f_x - Zf_r] t + \chi_{K+1} - \beta_{K+1Z}) \end{aligned} \quad (10)$$

Now calculate the time domain average by synchronizing, not with the rotation of the first gear, but with the fundamental gear tooth meshing frequency  $f_x$ . This is equivalent to multiplying the Fourier transform of the original signal by a comb filter comprising impulses located at multiples of  $f_x$ , thereby passing only components at these frequencies, including all the tooth meshing harmonics plus any sidebands which coincide with them. Whereas synchronizing with the gear rotation frequency produced the time domain average of the vibration for the complete gear, synchronizing with the fundamental tooth meshing frequency produces the time domain average of the vibration for a single tooth on that gear. In its simplest form, Stewart's enhancement technique eliminates the regular components which consist of the tooth meshing fundamental and harmonics from the time domain average of the vibration of the complete gear, these being the same components which form the time domain average of the meshing vibration for a single tooth. Stewart's technique thus subtracts the average meshing vibration from the original time domain average leaving the residual signal which describes the departures of the vibration from the average meshing vibration. The regular vibration does not necessarily equal the original unmodulated tooth meshing vibration, as the coincidence of modulation sidebands and tooth meshing harmonics as described in Equations 9 and 10 may change the regular signal.

The derivation so far has concentrated on the case when the phase modulation is small, allowing Equation 4 to be simplified to Equation 7. In practice it is possible that the phase modulation may be sufficiently large to invalidate the approximations used in these simplifications, particularly at the higher harmonics of the tooth meshing frequency, where a small error in tooth pitch will correspond to a larger phase error than at lower harmonics, or else when a local defect such as a fatigue crack is present in a gear. For example, a complete 360 degree phase lag in the vibration at the second harmonic of the tooth meshing frequency has been detected caused by an advanced fatigue crack in the input spiral bevel pinion in a helicopter gearbox [3]. If the phase modulation is large, the small angle approximations made previously will be invalid, and Equation 4 must be evaluated directly. For large deviations, phase modulation is



nonlinear, and resembles frequency modulation in that the modulation sidebands extend over a wide frequency range [6], making it more probable that some sidebands will coincide with adjacent meshing harmonics, even if  $N < 2$ . The derivation of a formal expression for  $c_k(t)$  when the phase modulation is large is tedious, and will not be attempted here. However, the effects of large phase modulation will be described qualitatively.

If a carrier signal is modulated by a single pure tone, the resulting signal can be expressed in terms of Bessel functions [6]. It can be shown that the modulation sidebands occur at multiples of the modulation frequency, and that there will be an infinite number of sidebands with the odd-order lower sidebands having reversed phase compared to the unmodulated carrier. The amplitude of the sidebands is dependent on the sideband order and on the maximum deviation of the modulation. For phase modulation by two pure tones which are not harmonically related, the result is more complicated, and consists of a carrier with modulation sidebands at multiples of both modulation frequencies, and also sidebands at the sum and difference frequencies of the modulating tones and their harmonics [6]. For modulation by multiple pure tones, there will be sidebands at all combinations of the sum and difference frequencies. For the gear system considered here, the modulating tones will all be multiples of the gear rotation frequency, and so the sum and difference frequencies will all reduce to multiples of the rotation frequency.

When amplitude modulation occurs simultaneously with large phase modulation, there will be modulation sidebands at the sum and difference frequencies of the modulating tones. For the gear system considered here, both amplitude and phase modulation will occur at harmonics of the gear rotation frequency, so that all sum and difference frequencies will reduce to multiples of the rotation frequency. Nevertheless, amplitude modulation with simultaneous phase modulation will change the pattern of the sidebands observed, and may modify the amplitudes of the meshing harmonics.

So far modulation has only been considered due to errors in one gear of a pair of meshing gears. In practice it is likely that both gears will contribute to the modulation of the meshing vibration due to variations in the tooth profile and pitch caused by manufacturing errors. Where both gears have significant phase deviations, sidebands at the sum and difference frequencies will be produced which will not in general coincide with the multiples of either rotation frequency, as mating gears are usually designed with the numbers of teeth having no common factor. Thus the time domain average which is calculated by synchronizing with the rotation of one gear will eliminate all but those components which coincide with the multiples of that gear rotation frequency. Nevertheless, it is possible that some sidebands will coincide with meshing harmonics, and in this manner modulation from one gear can be passed in the time domain average of the other gear. Even when the second gear is free of modulation, sidebands from the first gear can coincide with the meshing harmonics of the gear pair. If the time domain average of the second gear is calculated, then all components at multiples of that gear rotation frequency will be passed, including the sidebands from the first gear which coincide with the meshing harmonics. It is commonly thought that time domain averaging enables the complete elimination of the vibration of the other gear of a mating pair, provided that the numbers of teeth on the gears are not harmonically related. However, from the preceding discussion it can be seen this is not necessarily so, as modulation sidebands which coincide with meshing harmonics can also be passed by the time domain average of the other gear.



### 3. NUMERICAL EXAMPLES

The examples presented in this section were generated numerically using programs written in Fortran running on a DEC LSI-11 minicomputer under the RT-11 operating system. The signals were generated in blocks of 1024 samples, with one block corresponding to one revolution of a gear, using the inbuilt sine function, with a carrier frequency of 32 cycles per revolution, or 32 orders, and a nominal amplitude of unity. The data were not generated in the time domain and then averaged, but calculated directly, and so are exactly periodic. This is equivalent to calculating the time domain average using a very large number of averages. The results are presented graphically in four parts. Firstly, the original time signal over one revolution is given, and secondly, its amplitude spectrum. This is followed by the time signal reconstructed from the regular components comprising the harmonics of 32 orders, and finally the time signal showing the residual signal which is obtained after the regular signal is subtracted from the original. All the examples considered are simple and are intended to demonstrate the effects on the regular and residual signals of different modulation types, rather than to model the vibration of real gears which will be examined in the next section.

Figure 1a shows an unmodulated sine wave with a frequency of 32 orders and amplitude 1.0 units. The amplitude spectrum in Figure 1b shows a peak of amplitude 1.0 at 32 orders. The regular signal which is reconstructed from all the harmonics of 32 orders is shown in Figure 1c, and is identical to the original time signal. The residual signal, being the difference between the regular and original signals, is zero as shown in Figure 1d.

Figure 2a shows the sine wave after being amplitude modulated by another sine wave of frequency 1.0 orders and amplitude 0.5 units. Figure 2b shows the amplitude spectrum, which consists of a component at 32 orders having amplitude 1.0 units, straddled by modulation sidebands at a spacing of 1.0 orders and having amplitudes of 0.25 orders each. The regular signal in Figure 2c is identical to the unmodulated carrier. The residual signal in Figure 2d consists of oscillation at 32 orders having a maximum amplitude of 0.5 units. Careful examination of the residual shows that a phase reversal occurs at the nodes at 0.0, 0.5 and 1.0 revolutions. The frequency of the residual signal is 32 orders. It is thus possible to separate completely the regular and residual signals in this example.

Figure 3a shows the sine wave after being phase modulated by another sine wave of frequency 1.0 orders and amplitude 0.1 radians. The modulation is only small and is difficult to detect by inspection. The amplitude spectrum is shown in Figure 3b, consisting of a component at 32 orders with amplitude 1.0 units, straddled by modulation sidebands of amplitude 0.05 units each. Because the phase deviation is small, the modulation is approximately linear and only one pair of sidebands is produced. The regular signal in Figure 3c is practically identical to the unmodulated carrier, while the residual in Figure 3d consists of oscillation at a frequency of approximately 32 orders. Thus it is possible to separate completely the regular and residual signals for small phase modulation.



Figure 4a shows the sine wave after being phase modulated by another sine wave of frequency 1.0 orders but with an amplitude of 1.0 radians. The modulation is no longer small and can now just be discerned by inspection. The amplitude spectrum shown in Figure 4b consists of a component at 32 orders having an amplitude less than 1.0 units, surrounded by at least three pairs of modulation sidebands of decreasing amplitude. Because the phase deviation is no longer small, the small angle approximations are invalid and the phase modulation becomes a nonlinear process producing multiple sidebands about the carrier. The regular signal in Figure 4c has a lower amplitude than the unmodulated carrier because in phase modulation the carrier contains some of the modulation information, unlike amplitude modulation in which all of the information is contained in the sidebands [6]. The residual signal shown in Figure 4d consists of an oscillation at approximately 32 orders, but close examination reveals that the instantaneous frequency is lower at the nodes and higher at the antinodes, although no phase reversals occur at the nodes as was observed for amplitude modulation. The amplitude of the residual is less than 1.0. Thus the regular and residual signals cannot be completely separated when the phase modulation is large. This is an important result as large phase lags can be encountered in practice due to fatigue cracks in gears [3].

Figure 5a shows the sine wave after amplitude modulation by a rectangular pulse of frequency 1.0 orders and amplitude 0.9 units with a pulse duration of two cycles, or 1/16 revolutions. The amplitude spectrum is shown in Figure 5b, consisting of a component at 32 orders of amplitude just greater than 1.0 units, surrounded by a band of low level modulation sidebands extending beyond the adjacent meshing harmonics. The carrier amplitude is greater than 1.0 because the energy level of the original signal has been increased slightly by the pulse modulation. The regular signal in Figure 5c has an amplitude slightly greater than 1.0 for the same reason. The residual signal shown in Figure 5d consists of oscillation at an amplitude just less than 0.9 units for a duration of two cycles, then a low level oscillation over the remainder of the signal at a frequency of 32 orders but out of phase with the initial cycles. The original signal can be viewed as the sum of the carrier at a constant amplitude and a short burst at the same frequency. The spectrum of the burst alone features a component at 32 orders but with low amplitude, surrounded by low level sidebands. The amplitudes of the components at harmonics of 32 orders will be very low. The regular signal is the sum of the unmodulated carrier and the components at harmonics of 32 orders, and so has slightly greater amplitude. The residual signal is the same as the short burst but with the components at harmonics of 32 removed, and so has the form of a burst with a low level signal at 32 orders removed from it. This explains the phase difference observed. This feature has not been observed in the previous example of amplitude modulation because sinusoidal modulation did not modify the average amplitude of the carrier.

For the simple cases of amplitude modulation with no change of average carrier amplitude and of phase modulation with small deviation with no change of carrier amplitude, there is no coincidence between the modulation sidebands and the adjacent meshing harmonics. This enables the complete separation of the modulation from the carrier and is indicated by the regular signal having the same amplitude as the unmodulated carrier. For large deviations, phase modulation is nonlinear and so produces many sidebands and also modifies the amplitude of the carrier in the spectrum. Modulation sidebands are likely to coincide with adjacent meshing harmonics so that the regular signal will have a form or amplitude different from the unmodulated carrier. The residual signal will also be different from the



modulating signal. For amplitude modulation by a signal with a nonzero average value the component at the carrier frequency will be added to the carrier and thus change the regular signal, leaving a ripple at the carrier frequency in the residual. That is, if the application of modulation to the carrier produces significant modulation sidebands at any of the tooth meshing harmonics, the regular signal will differ from the unmodulated carrier.

#### 4. PRACTICAL EXAMPLES

The two examples in this section are taken from in-flight recordings of the vibration of the main rotor gearbox of a helicopter with a fatigue crack at the root of one of the teeth of the input spiral bevel pinion gear [7]. The gear has 22 teeth and rotates at approximately 43 Hz, transmitting about 1 MW at full load. The vibration was sensed by an accelerometer mounted on the gearbox housing near the main taper roller bearing which supports the gear. In the laboratory, time domain averages were calculated from the recordings using a DEC LSI-11 minicomputer running under the RT-11 operating system, with blocks of 512 samples being produced after 256 or more averages. The results are presented in the same form as for the numerically generated examples in the previous section.

The time domain average obtained when the gear had an early fatigue crack is shown in Figure 6a. Note the location of the change in the vibration caused by the crack cannot be readily identified in the time domain average without prior knowledge. There are some variations in the amplitude of the vibration, but there is nothing unusual to be seen in the signal compared with the time domain average of the vibration of other gears of the same type in good condition. The amplitude spectrum shown in Figure 6b indicates a large peak at 44 orders, being the second harmonic of the tooth meshing frequency which predominates in this particular gearbox. There are also some small modulation sidebands about this component. The regular signal in Figure 6c is reconstructed from all of the harmonics of 22 shaft orders, and it features 44 cycles per revolution, but with every second cycle having a slightly larger amplitude. The residual signal in Figure 6d shows an apparently random pattern due to the normal variations in tooth profile and pitch, except at the centre of the trace where a slightly larger transient oscillation is evident, caused by the fatigue crack. The oscillation at 44 orders persists over several cycles, yet there is no pronounced change in amplitude of the original signal at the same location because the residual signal is due to phase modulation.

When the fatigue crack in the gear had grown to an advanced stage, the time domain average shown in Figure 7a was obtained. It is now possible to identify clearly the change in the vibration which is produced by the crack as an abrupt negative peak near the centre of the trace, followed by a transient and then a return to the normal meshing pattern. The amplitude spectrum in Figure 7b shows a peak at 44 orders but of lower amplitude than was observed for the early crack. However, the second average was calculated from a recording made at a lower torque which will influence the amplitude of the meshing vibration. The modulation sidebands which are immediately adjacent to the peak at 44 orders are not much larger than for the early crack, but similar sidebands also occur about the harmonic at 66 orders. In addition, there are clusters of components centred at 18 and 36 orders which are believed to be caused by resonances in the transfer path



between the gear teeth and the location of the transducer being excited by the abrupt load changes which occur as the crack goes the mesh [4]. The regular components in Figure 7c are similar in appearance to those in Figure 6c, except for the slightly lower amplitude which may be due to the lower torque. The residual signal in Figure 7d shows random variations for a part of the trace, then a broad transient oscillation of greater amplitude and duration than for the early crack.

For both the early and advanced fatigue cracks the regular signal has a similar appearance, although the amplitudes of the signals cannot be directly compared because of the different torque. For the early fatigue crack, the modulation is small and confined mainly to the harmonic at 44 orders. Nevertheless there is a broad spread of very low level sidebands which are probably due to the normal variations in tooth pitch and profile. There is likely to be some coincidence between modulation sidebands and meshing harmonics, but this cannot be observed because the effect would only be small as the modulation is small. For the advanced crack, there is more extensive modulation as well as excitation of resonances, but it is still not possible to quantify changes which are due to coincidence.

While the presence of the early crack may be deduced from the residual signal in Figure 6d, it is arguably not as clear to the analyst as the evidence provided by other methods [2,3] of enhancement, probably because of the random variations which occur in the normal part of the trace due to pitch and profile variations. The residual signal contains broad band information which includes these variations, even though the information describing the crack is concentrated in the modulation sidebands about the strong meshing harmonics. Narrow band techniques eliminate much of the pitch and profile variation signal, and so in principle should give better discrimination. The advanced crack may be easily distinguished in the residual signal, but is also detected readily in the original time domain average without the need of enhancement. Even so, demodulation of the time domain average gives a clearer indication of a crack than does the residual [3]. So for these examples it would appear that the narrow band enhancement techniques may enable easier detection of fatigue cracks in gear teeth, largely due to the occurrence of a phase lag in the vibration produced by the affected teeth. This finding may not necessarily apply to other failure modes.

## 5. CONCLUSIONS

This technical memorandum has examined an existing technique for the signal processing of the time domain average of the vibration produced by meshing gears with application to the early detection of gear failure. It has been shown that the regular signal consisting of the tooth meshing harmonics which can be extracted from the time domain average of the vibration of a complete gear actually defines the time domain average of the meshing vibration of a single gear tooth. The residual signal, which is obtained by subtracting the regular signal from the original time domain average, gives the departures of the vibration from the average. The regular vibration does not necessarily represent the original unmodulated tooth meshing vibration, as the coincidence of modulation sidebands and tooth meshing harmonics may change the regular signal.



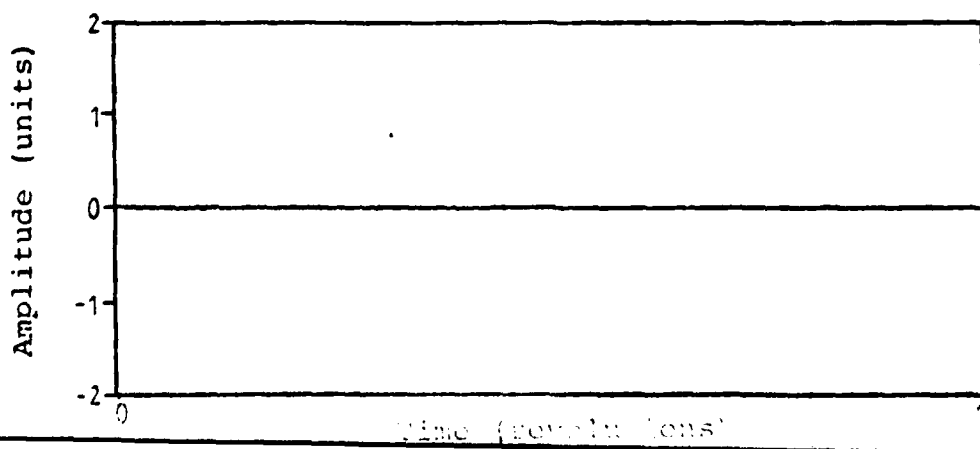
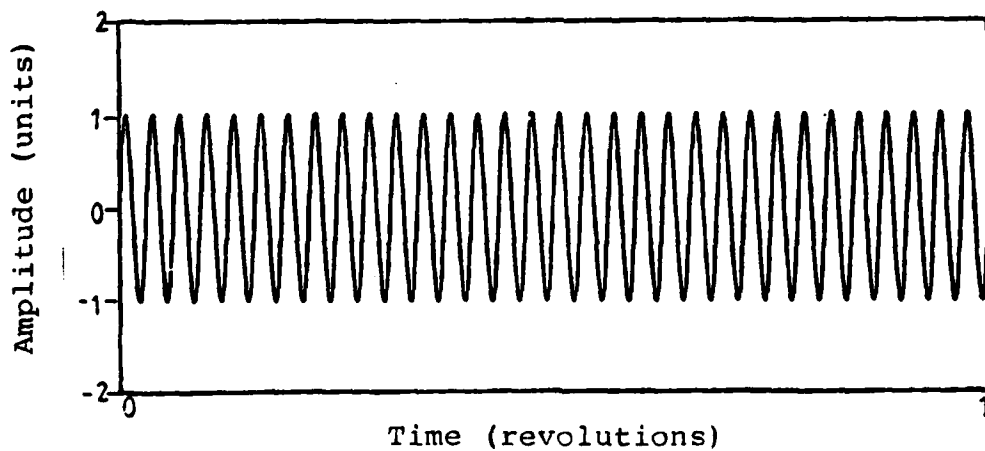
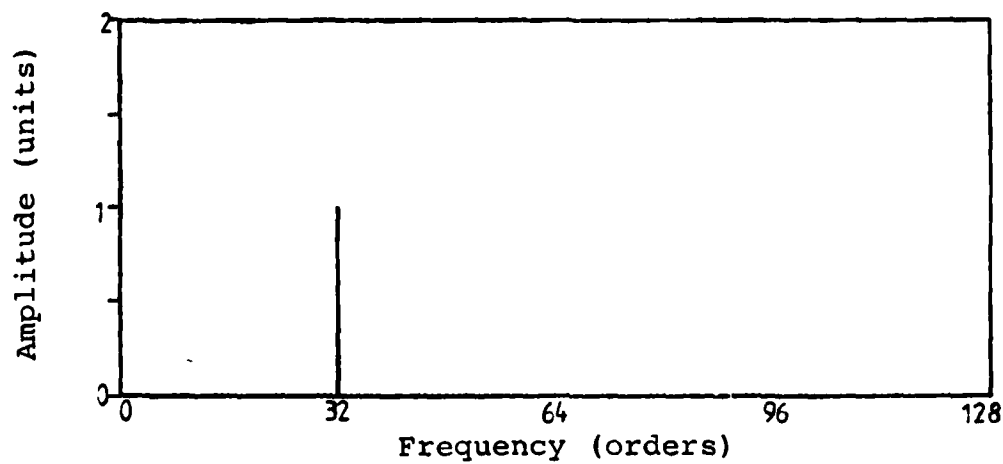
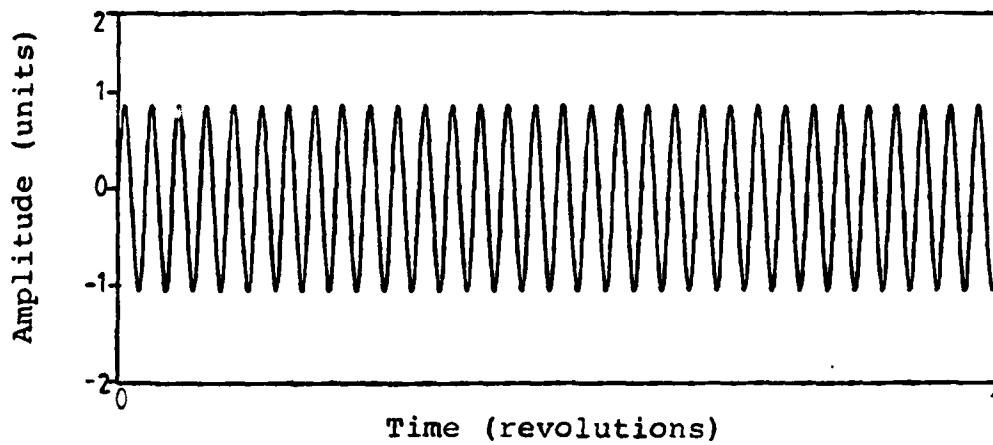
Simple numerically generated examples have been examined, based on a sinusoidal carrier and a variety of simple modulation schemes. The difference between small and large deviation phase modulation has been demonstrated, as well as the effects of the coincidence between harmonics and sidebands. Practical examples have been examined, based on the vibration recordings of a helicopter gearbox with a fatigue crack in the input bevel pinion gear. While the residual signal does enable the crack to be detected, the broad band nature of the technique passes residual signals due to normal gear tooth pitch and profile variations, which could make detection less sensitive in these examples than narrow band techniques which eliminate much of this background.



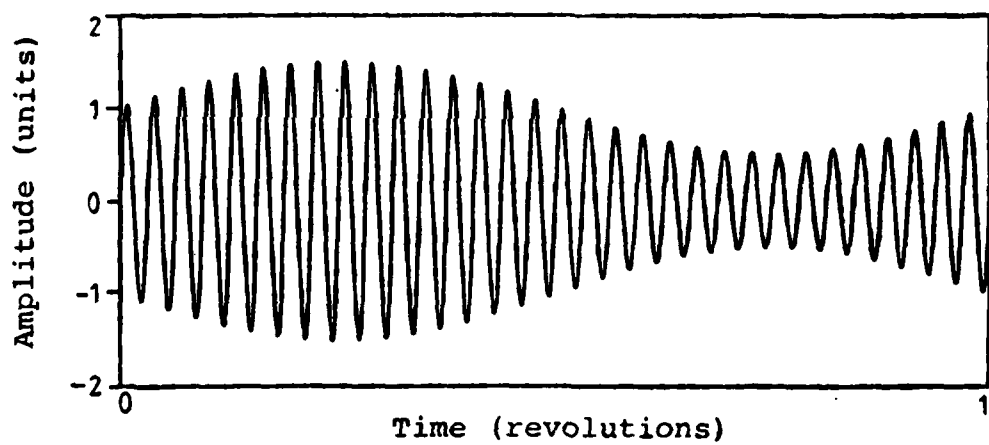
## REFERENCES

- 1) Stewart, R.M., "Some Useful Data Analysis Techniques for Gearbox Diagnostics", ISVR, University of Southampton, MHM/R/10/77, July 1977.
- 2) McFadden, P.D. and Smith, J.D., "A Signal Processing Technique for Detecting Local Defects in a Gear from the Signal Average of the Vibration", Proceedings of the Institution of Mechanical Engineers, vol 199, part C, no 4, pp 287-292, 1985.
- 3) McFadden, P.D., "Detecting Fatigue Cracks in Gears by Amplitude and Phase Demodulation of the Meshing Vibration", Transactions of the American Society of Mechanical Engineers, Journal of Vibration Acoustics Stress and Reliability in Design, to be published.
- 4) McFadden, P.D., "Low Frequency Vibration Generated by Gear Tooth Impacts", NDT international, vol 18, no 5, pp 279-282, October 1985.
- 5) Braun, S., "The Extraction of Periodic Waveforms by Time Domain Averaging", Acustica, vol 32, pp 69-77, 1975.
- 6) Carlson, A.B., "Communication Systems", McGraw-Hill, 2nd edition, pp 231-232, 224-229, 240-241, 1975.
- 7) McFadden, P.D., "Analysis of the Vibration of the Input Bevel Pinion in RAN Wessex Helicopter Main Rotor Gearbox WAK143 Prior to Failure", Aeronautical Research Laboratories, Aero Propulsion Report 169, September 1985.

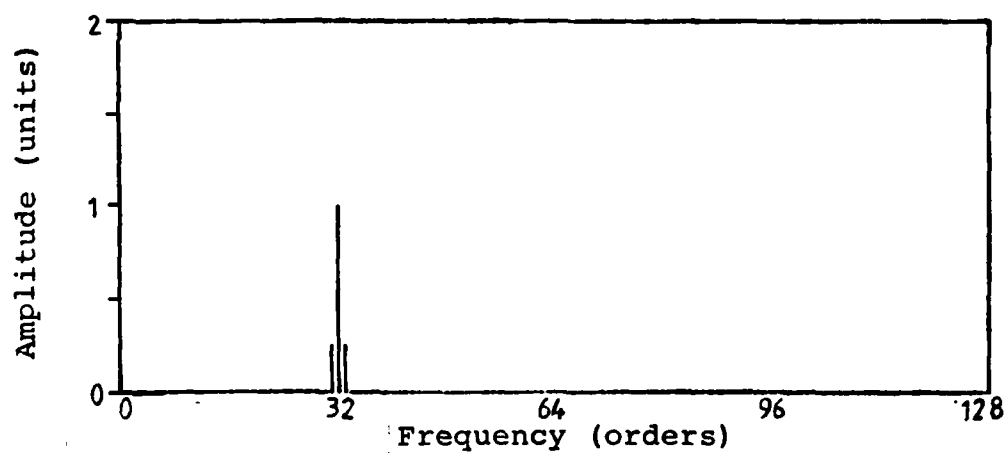




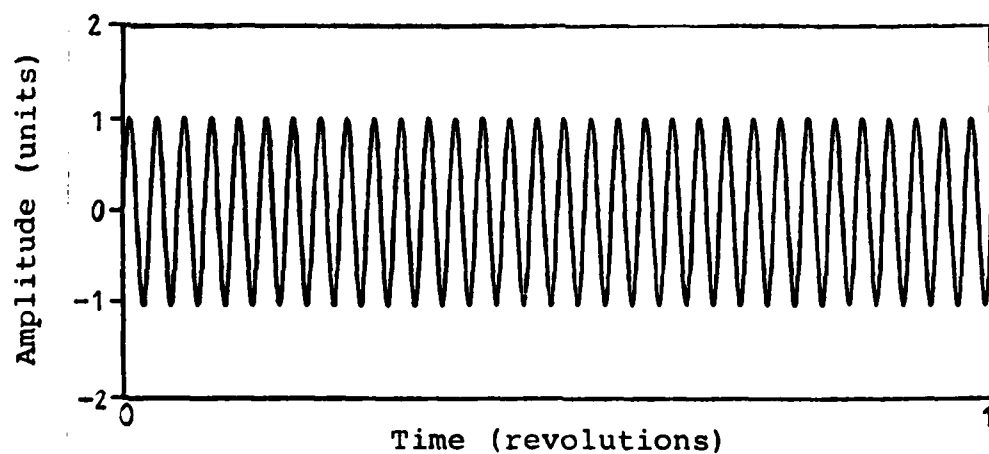




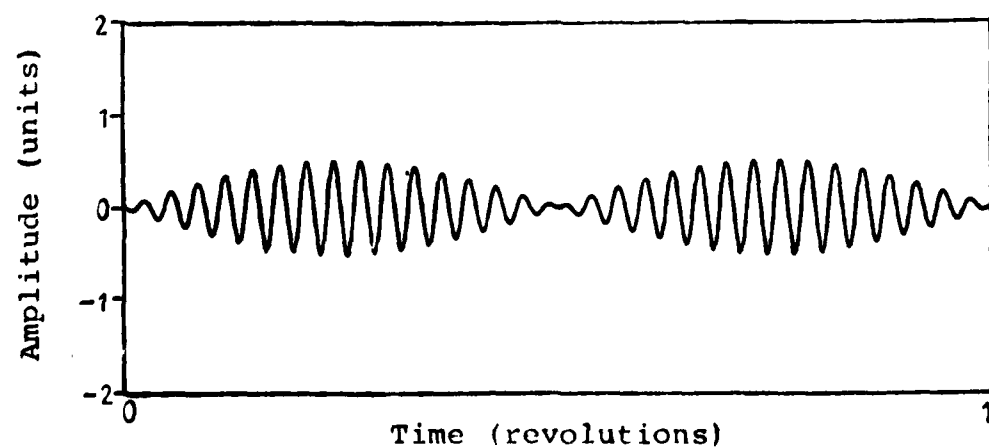
(a)



(b)



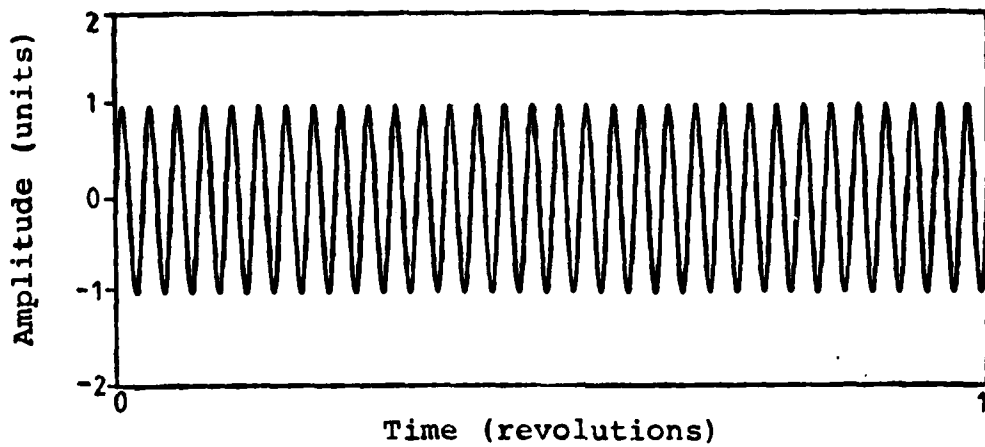
(c)



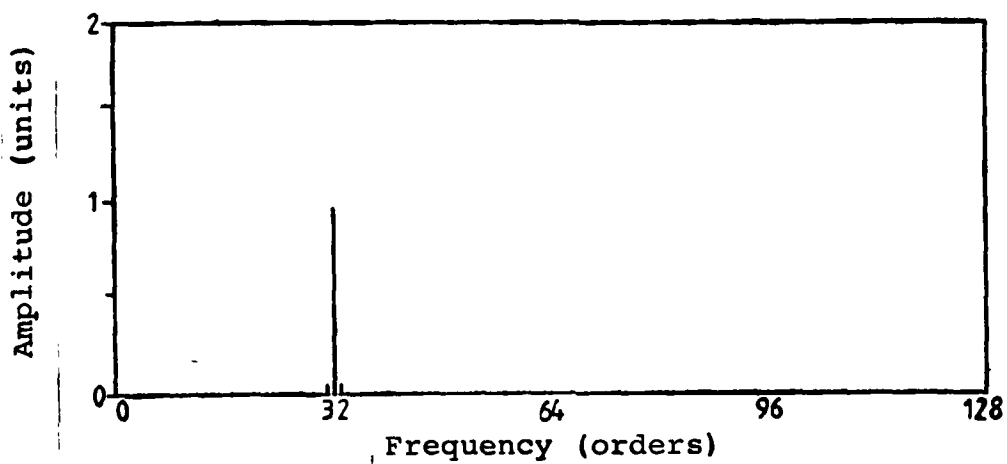
(d)

FIG. 2

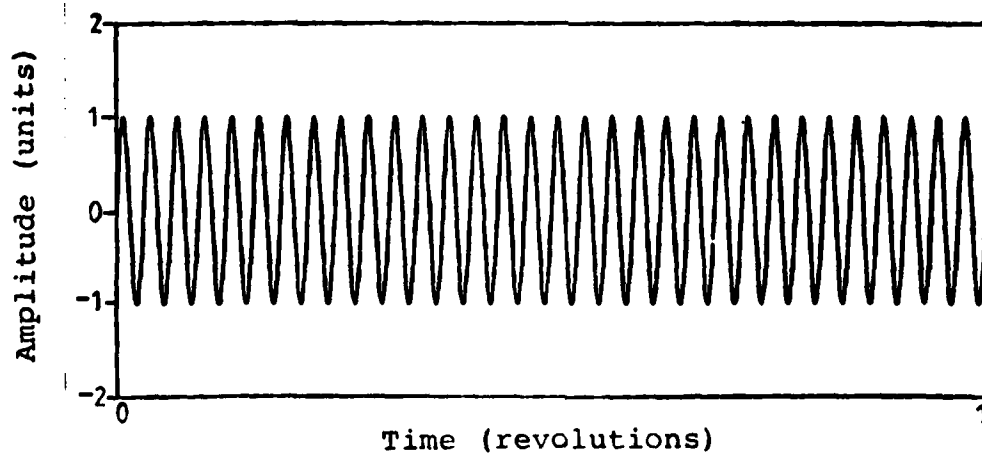




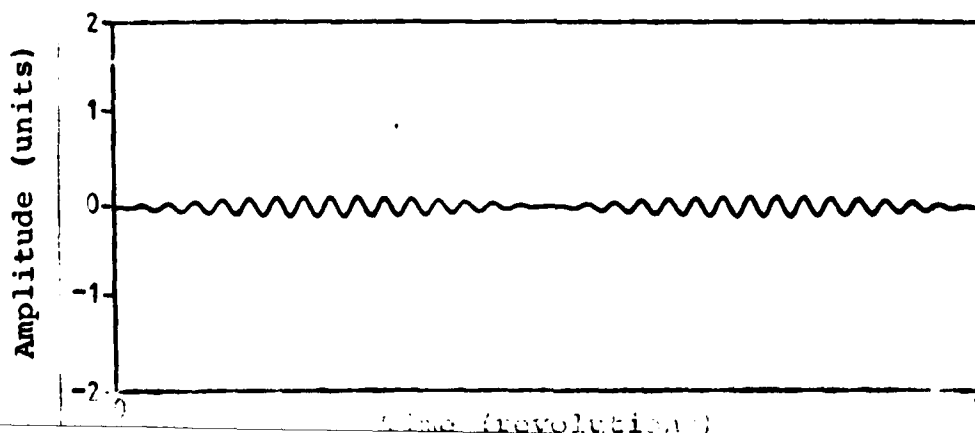
(a)



(b)

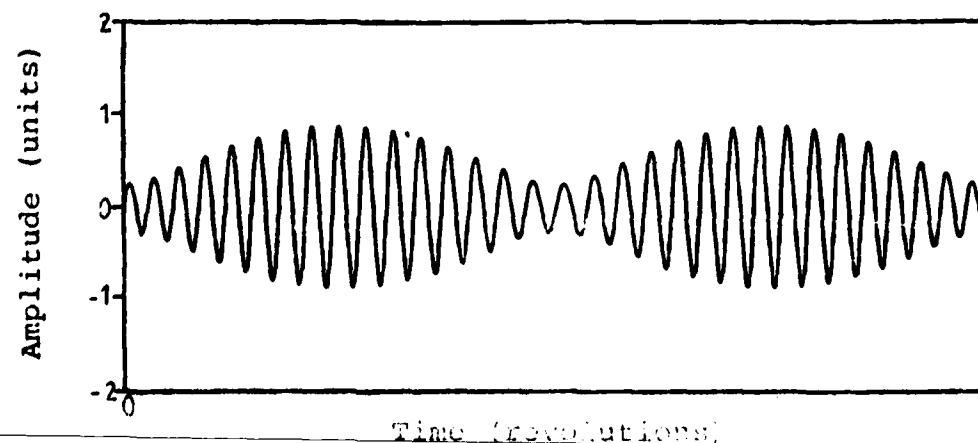
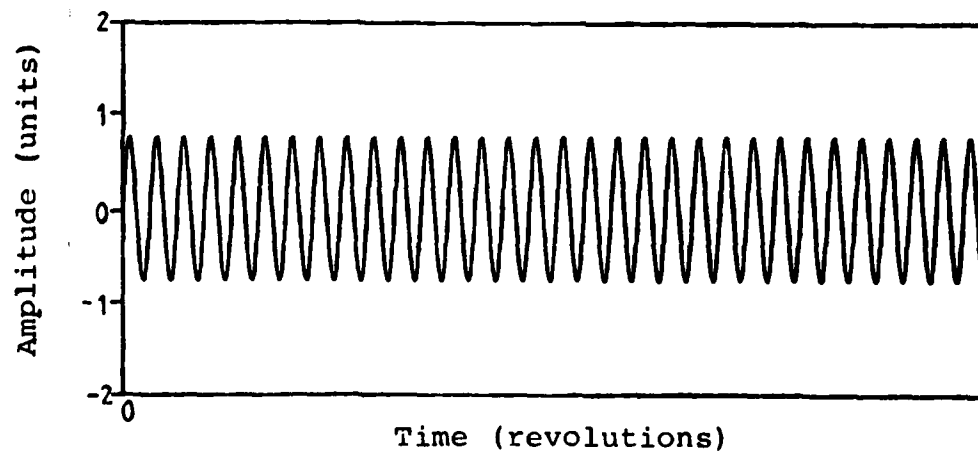
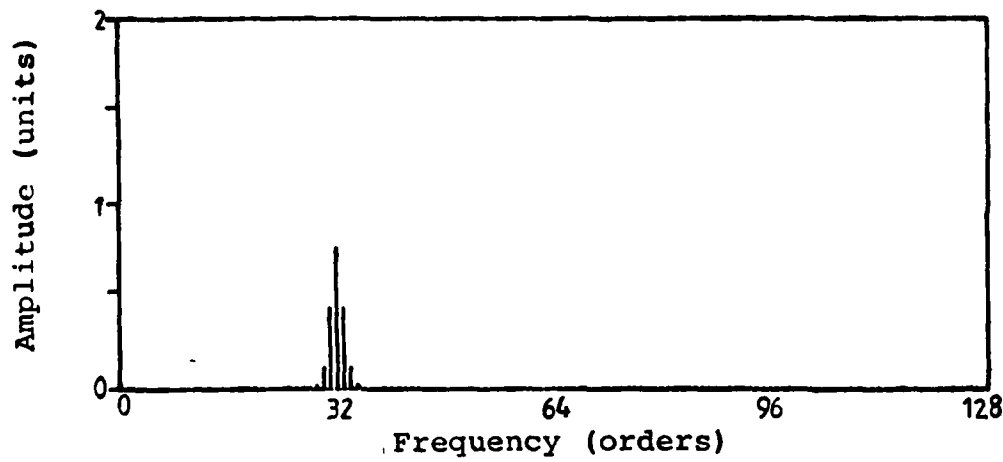
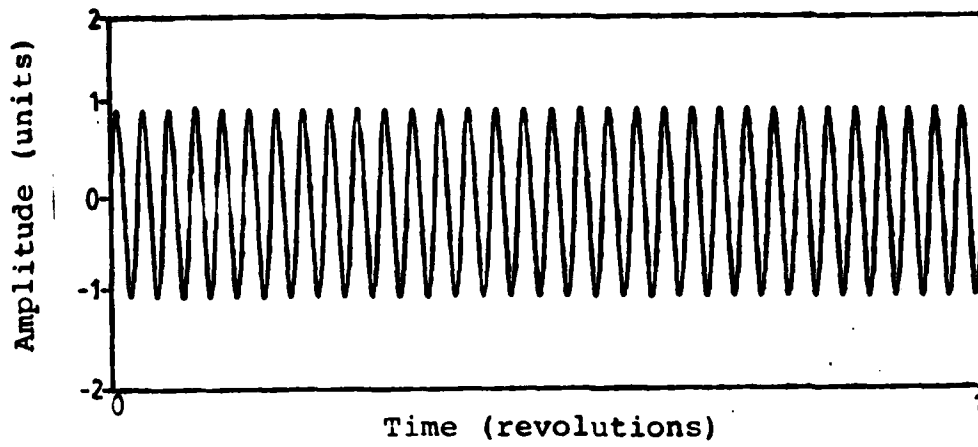


(c)

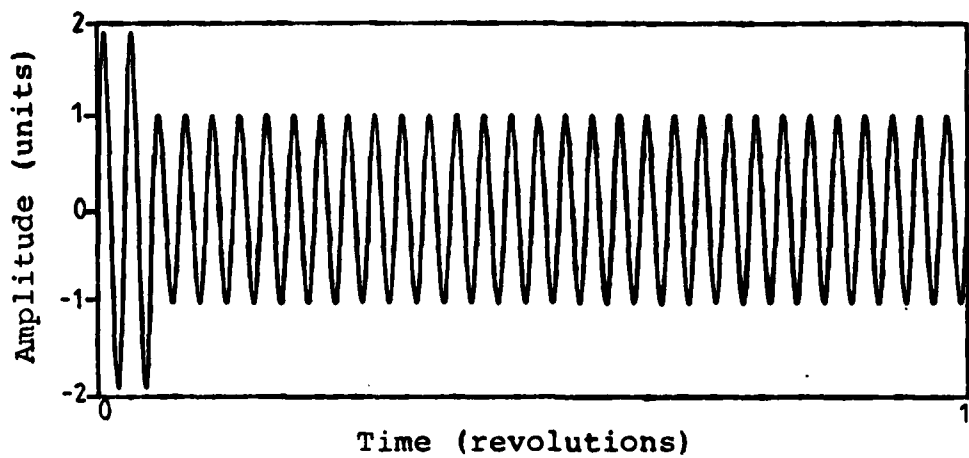


(d)

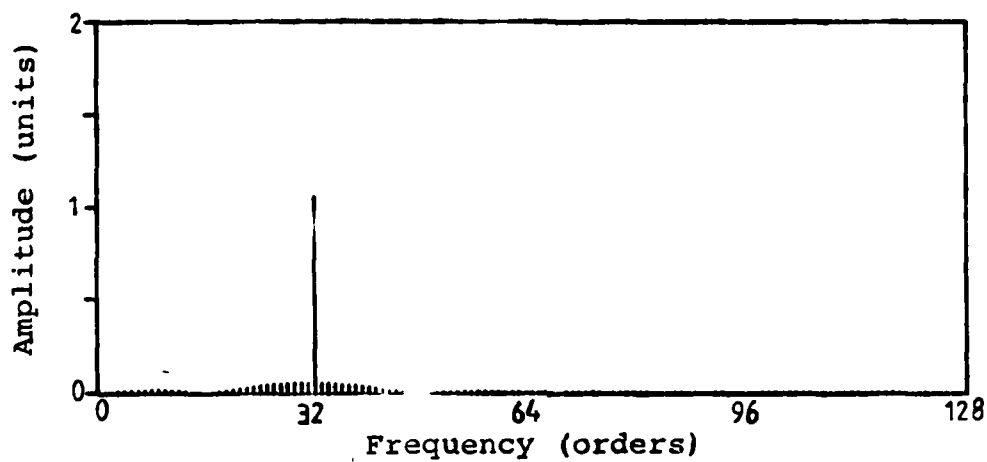




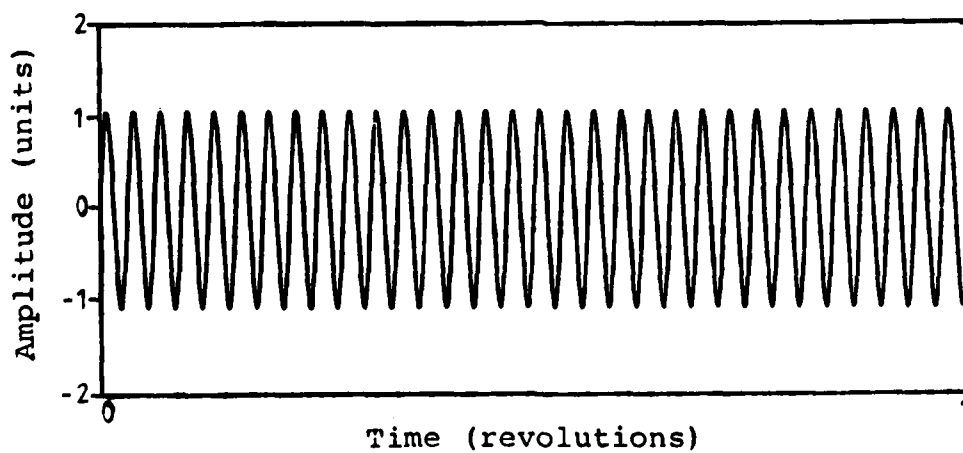




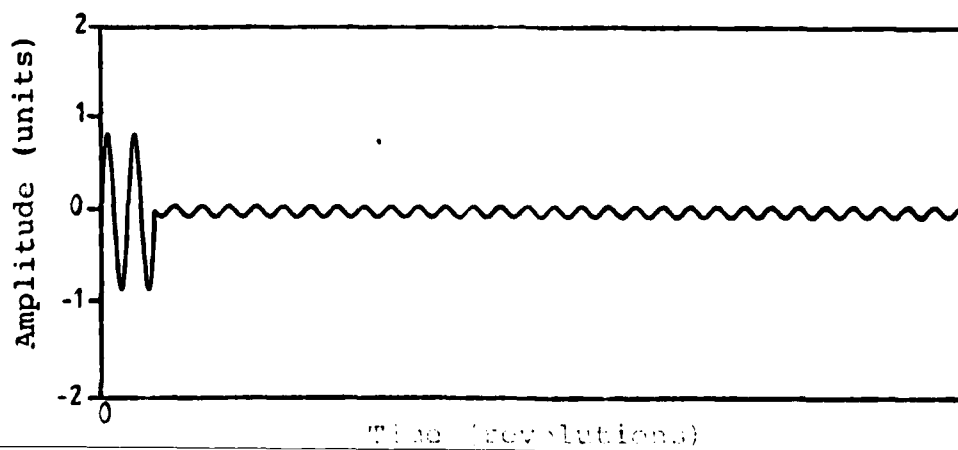
(a)



(b)

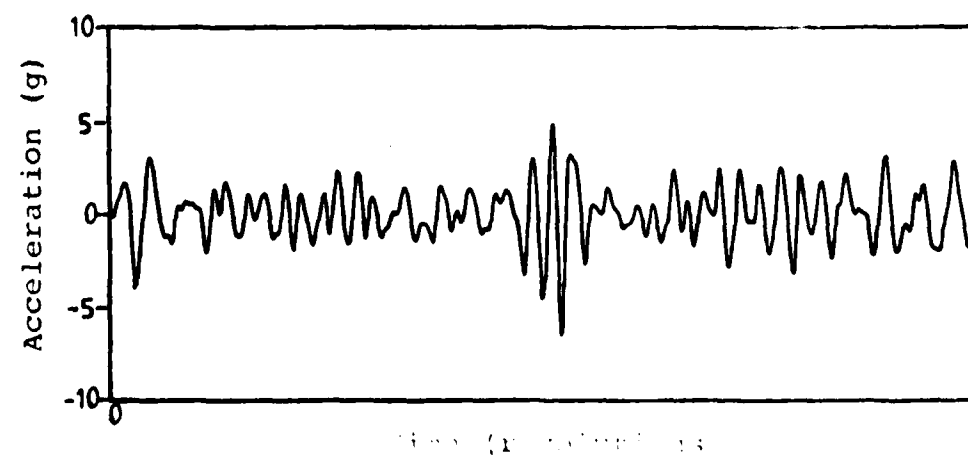
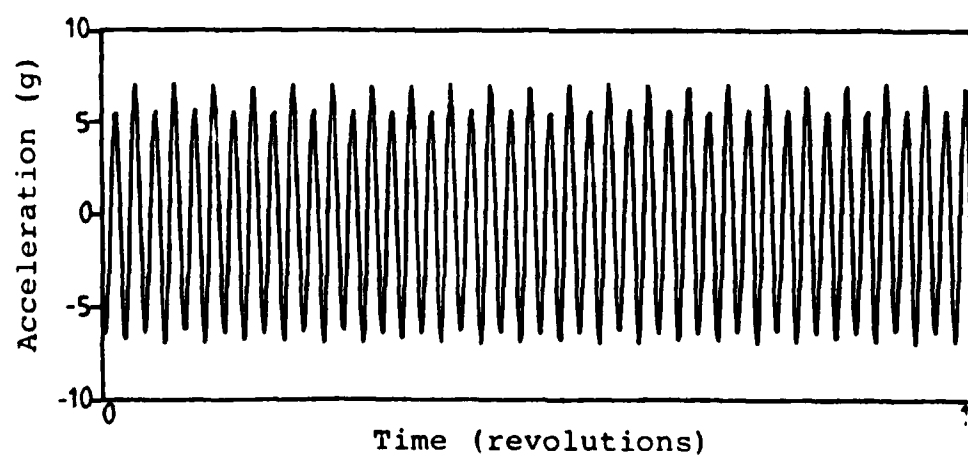
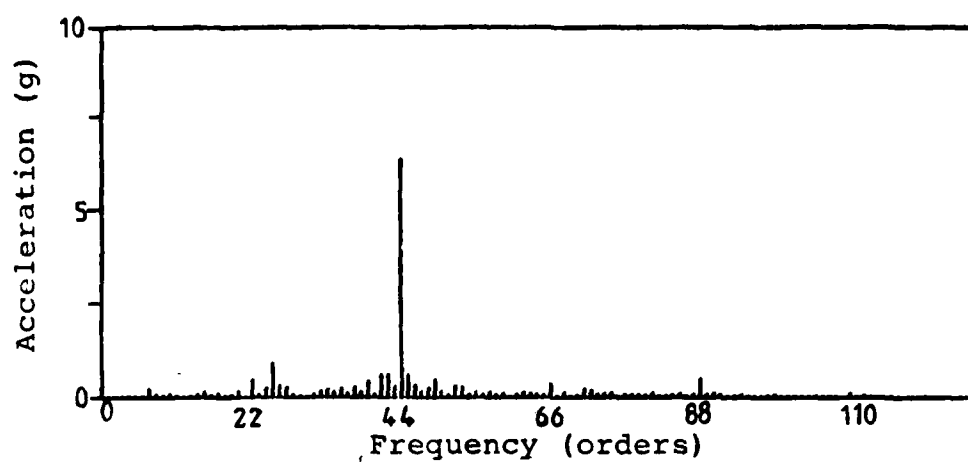
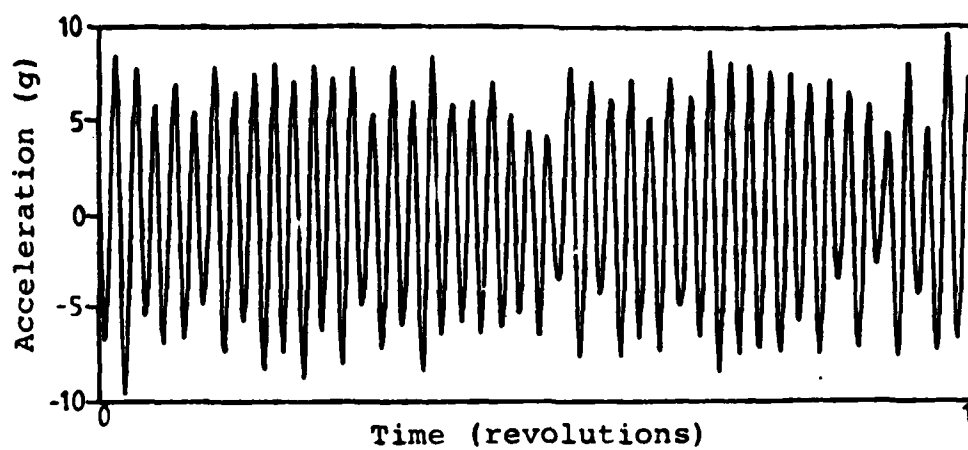


(c)

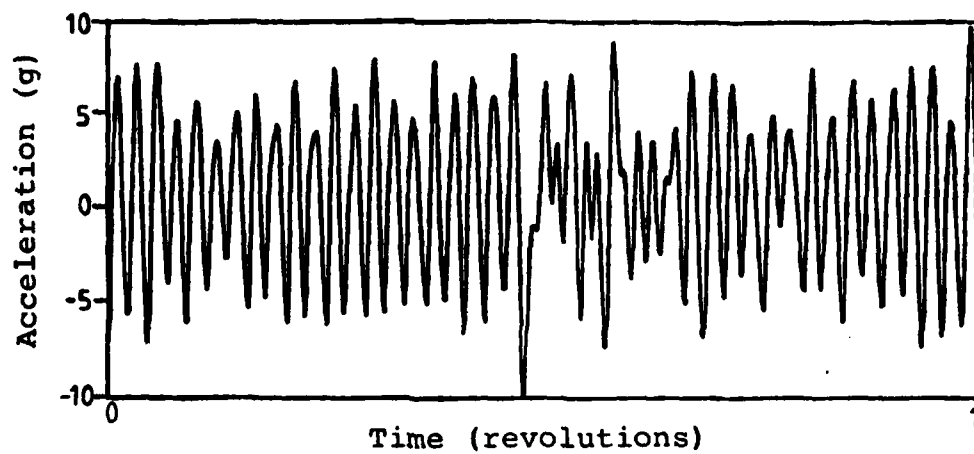


(d)

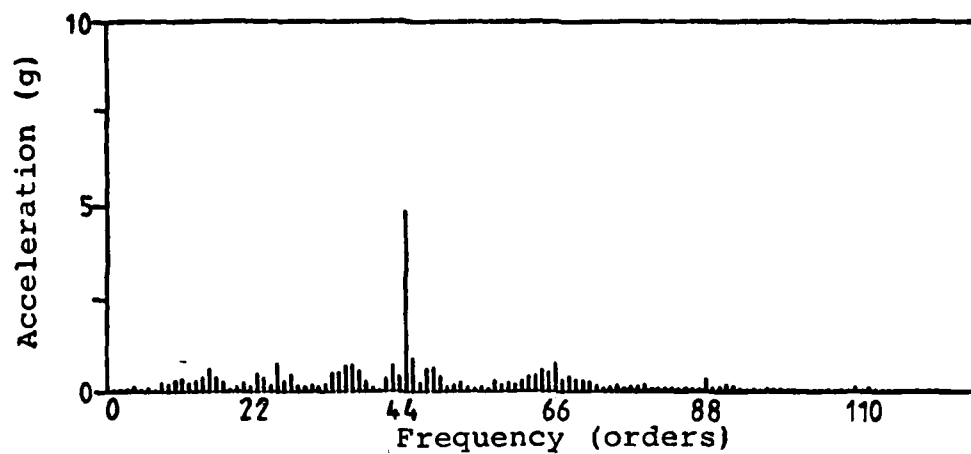




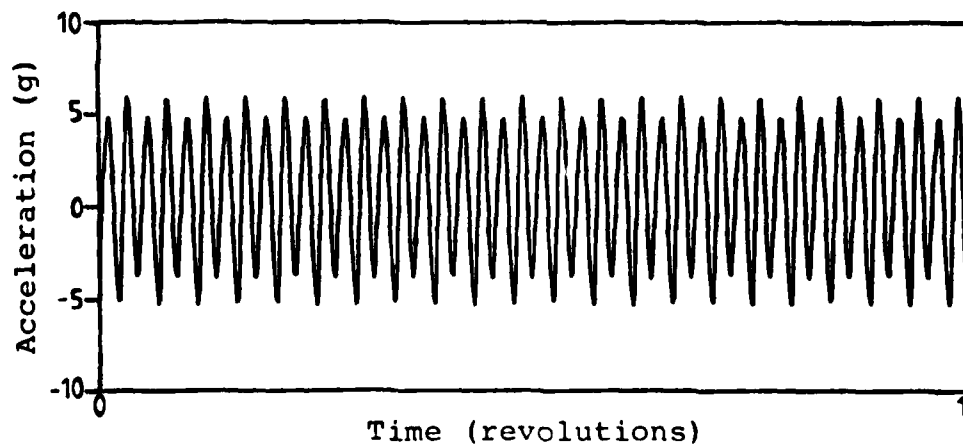




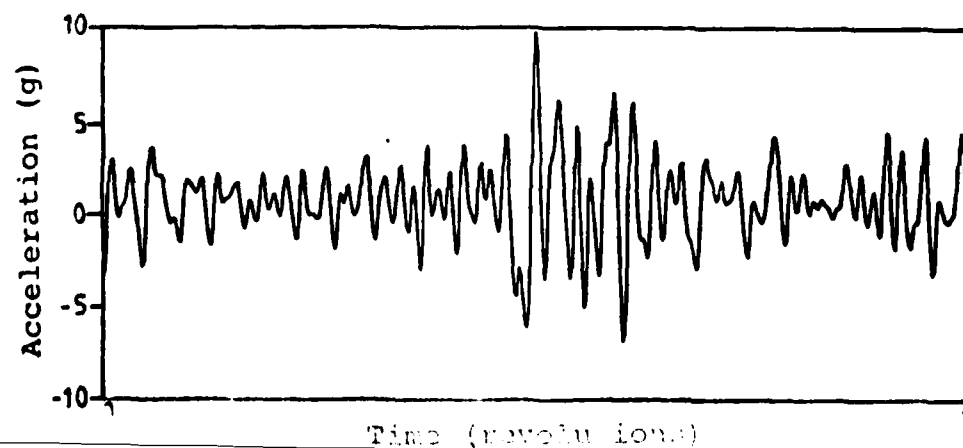
(a)



(b)



(c)



(d)



## DISTRIBUTION

### AUSTRALIA

#### Department of Defence

##### Defence Central

Chief Defence Scientist )  
Deputy Chief Defence Scientist )  
Superintendent, Science and Program Administration ) 1 copy  
Controller, External Relations, Projects and )  
Analytical Studies )  
Defence Science Adviser (UK) (Doc Data sheet only)  
Counsellor, Defence Science (USA) (Doc Data sheet only)  
Defence Science Representative (Bangkok)  
Defence Central Library  
Document Exchange Centre, DISB (18 copies)  
Joint Intelligence Organisation  
Librarian H Block, Victoria Barracks, Melbourne  
Director General - Army Development (NSO) (4 copies)

##### Aeronautical Research Laboratories

Director  
Library  
Superintendent - Aero Propulsion  
Divisional File - Aero Propulsion  
Author: P.D. McFadden

##### Materials Research Laboratories

Director/Library

##### Defence Research Centre

Library

##### Navy Office

Navy Scientific Adviser  
RAN Aircraft Maintenance and Flight Trials Unit  
Directorate of Naval Aircraft Engineering  
Directorate of Naval Aviation Policy  
Superintendent, Aircraft Maintenance and Repair

##### Army Office

Scientific Adviser - Army  
Engineering Development Establishment, Library  
Royal Military College Library

.../cont.



## DISTRIBUTION (cont.)

### Air Force Office

Air Force Scientific Adviser  
Aircraft Research and Development Unit  
Scientific Flight Group  
Library  
Technical Division Library

### Government Aircraft Factories

Library

### Department of Aviation

Library  
Flight Standards Division

### Industry

Commonwealth Aircraft Corporation, Library  
Hawker de Havilland Aust. Pty Ltd, Bankstown, Library

### Universities and Colleges

Adelaide	Barr Smith Library
Flinders	Library
La Trobe	Library
Melbourne	Engineering Library
Monash	Hargrave Library
Newcastle	Library
Sydney	Engineering Library
NSW	Physical Sciences Library
Queensland	Library
Tasmania	Engineering Library
Western Australia	Library
RMIT	Library

### UNITED KINGDOM

Naval Aircraft Materials Laboratory  
Library  
Stewart Hughes Limited, Southampton  
Westland Helicopters Limited, Yeovil  
Library

**SPARES (10 copies)**

**TOTAL (77 copies)**



## DOCUMENT CONTROL DATA

1. a. AR No AR-004-455	1. b. Establishment No ARL-AERO-PROP-TM-434	2. Document Date APRIL 1986	3. Task No DST 82/051
4. Title EXAMINATION OF A TECHNIQUE FOR THE EARLY DETECTION OF FAILURE IN GEARS BY SIGNAL PROCESSING OF THE TIME DOMAIN AVERAGE OF THE MESHING VIBRATION		5. Security a. document UNCLASSIFIED	6. No Pages 17
		b. title U	c. abstract U
7. No Refs 7			
8. Author(s)  P.D. McFADDEN		9. Downgrading Instructions  —	
10. Corporate Author and Address  Aeronautical Research Laboratories PO Box 4331 MELBOURNE VIC 3001		11. Authority (as appropriate) a. Sponsor b. Security c. Downgrading d. Approval  —	
12. Secondary Distribution (of this document)  Approved for Public Release			
Overseas enquirers outside stated limitations should be referred through ASDIS, Defence Information Services Branch, Department of Defence, Campbell Park, CANBERRA ACT 2601			
13. a. This document may be ANNOUNCED in catalogues and awareness services available to ... No limitations			
13. b. Citation for other purposes (ie casual announcement) may be (select) unrestricted (or) as for 13 a.			
14. Descriptors  Vibration Gearboxes Signal averaging Condition monitoring		15. COSATI Group  01030 13090 14020	
18. Abstract  An existing technique for the signal processing of the time domain average of the tooth meshing vibration of gears is examined with application to the early detection of failure. It is shown that the "regular" signal extracted from the time domain average of the gear vibration by the elimination of the fundamental and harmonics of the tooth meshing frequency forms the time domain average of the vibration of a single tooth. The "residual" signal which is obtained by subtracting the regular signal from the original time domain average gives the departures of the vibration from the average. Numerical and practical examples are given.			



This page is to be used to record information which is required by the Establishment for its own use but which will not be added to the DISTIS data base unless specifically requested.

16. Abstract (Contd)		
17. Imprint Aeronautical Research Laboratories, Melbourne		
18. Document Series and Number Aero Propulsion Technical Memorandum 434	19. Cost Code 41 7425	20. Type of Report and Period Covered
21. Computer Programs Used		
22. Establishment File Ref(s)		



END

DTIC

9-86